

The Role of 5-quark Components on the Nucleon Form Factors

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Abstract

The covariant quark model is shown to allow a phenomenological description of the neutron electric form factor, $G_E^n(Q^2)$, in the impulse approximation, provided that the wave function contains minor ($\sim 3\%$) admixtures of the lowest energy sea-quark configurations. While that form factor is not very sensitive to whether the \bar{q} in the $qqqq\bar{q}$ component is in the P -state or in the S -state the calculated nucleon magnetic form factors are much closer to the empirical values in the case of the former configuration. In the case of the electric form factor of the proton, $G_E^p(Q^2)$ a zero appears in the impulse approximation close to 10 GeV^2 , when the \bar{q} is in the P -state. That configuration, which may be interpreted as a pion loop (“cloud”) fluctuation, also leads to a clearly better description of the nucleon magnetic moments. When the amplitude of the sea-quark admixtures are set so as to describe the electric form factor of the neutron, the $qqqq\bar{q}$ admixtures have the phenomenologically desirable feature, that the electric form factor of the proton falls at a more rapid rate with momentum transfer than the magnetic form factor.

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I. INTRODUCTION

Phenomenological analyses of the electric form factor of the neutron have indicated that the neutron may be viewed as a 3-quark core surrounded by a meson cloud [1]. This result is qualitatively consistent with hadronic model studies of the electromagnetic and weak decay patterns of the $\Delta(1232)$ resonance, which show that the phenomenological failures of the conventional qqq model of the nucleon and the resonance may be understood as a consequences of the lack of a meson cloud component in the $\Delta(1232)$ [2, 3]. While the covariant extensions of the qqq quark model may be tuned to provide a qualitatively satisfactory description of the extant data on the nucleon form factors, including that on the electric form factor of the neutron [4], it is nevertheless natural to extend the quark model to include explicit sea-quark contributions [5], which more directly may be interpreted as meson cloud components. This is so much more the case as several measurements of the \bar{d}/\bar{u} asymmetry in the nucleon sea indicate the presence of $qqqq\bar{q}$ components in the proton [6, 7, 8, 9].

Here a calculation of the contribution of $qqqq\bar{q}$ components in the nucleon wave functions to their electromagnetic form factors is described in instant form kinematics. In view of the large number of possible $qqqq\bar{q}$ configurations [10], the calculation is restricted to those $qqqq\bar{q}$ configurations that are expected to have the lowest energy with the \bar{q} either in the excited P -state or in the S -state. In the latter case positive parity requires the $qqqq$ subsystem to be in the P -state. The calculation is carried out with the covariant quark model with instant form kinematics, which is similar to the nonrelativistic quark model, save for the explicit treatment of the constituent boosts.

The spatial wave function is taken to have a simple algebraic form with two parameters, which are determined by fits to the electric form factors of the nucleons. The amplitude of the $qqqq\bar{q}$ component in the wave function is determined by the fit to the electric form factor of the neutron. The $qqqq\bar{q}$ contributions to the form factors do depend on the radial part of the wave function, but with little sensitivity to whether the \bar{q} is in the S -state or in the P -state. In the case of the electric form factor of the proton, $G_E^p(Q^2)$, the inclusion of the $qqqq\bar{q}$ component markedly improves the description of the most recent data, which shows that the electric form factor falls much faster with momentum transfer than the corresponding magnetic form factor. In case where the antiquark is in the P -state, the impulse approximation leads to a zero in the calculated electric form factor between 10 and

11 GeV². In this case the quantum numbers of the $qqqq\bar{q}$ configuration correspond to those of a pion loop fluctuation, and hence admit an interpretation as a “pion cloud” configuration. This configuration is the preferred one for the description of the nucleon magnetic moments.

The nucleon wave functions in the extended quark model are described in section 2, along with the boosts to the Breit frame that are needed in instant form kinematics. The calculation of the electromagnetic form factors of the neutron and the proton is described in section 3. Section 4 contains a summarizing discussion.

II. NUCLEON WAVE FUNCTIONS WITH 3 AND 5 QUARKS

A. The qqq component

The radial wave functions of the nucleons formed by n constituents in their rest frame may be expressed in terms of the following set of relative momenta:

$$\vec{\xi}_j = \frac{1}{\sqrt{j+j^2}} \left[\sum_{l=1}^j \vec{k}_l - j\vec{k}_{j+1} \right], \quad j = 1, \dots, n-1. \quad (1)$$

The rest frame is defined by the condition that $\sum_{i=1}^n \vec{k}_i = 0$.

In the case of the qqq component, where all the constituent quarks are in the ground state, the radial wave function will be taken to have the form [4]:

$$\varphi(\vec{\xi}_1, \vec{\xi}_2) = \mathcal{N}_3 \frac{1}{(1 + (\frac{\vec{\xi}_1^2 + \vec{\xi}_2^2}{2b^2}))^a}. \quad (2)$$

Here \mathcal{N}_3 is a normalization constant. The two parameters a and b may be determined by a fit to the empirical proton form factor.

The spin-isospin part of the qqq component shall be taken to have the usual $SU(2)$ symmetric form, which is formed by the symmetric combination of mixed symmetry spin and isospin wave functions: $[3]_{FS}[21]_F[21]_S$ (Here F stands for flavor and S for spin). The complete wave function for the nucleon may then be expressed in the compact form:

$$\psi_N(t, s) = \psi_{[111]}^C \varphi(\vec{\xi}_1, \vec{\xi}_2) \sum_a C_{[21]_a, [21]_a}^{[3]} \psi_{[21]_a}^F(t) \psi_{[21]_a}^S(s). \quad (3)$$

Here the antisymmetric color component is denoted $\psi_{[111]}^C$ and the mixed symmetry isospin and spin components are denoted $\psi_{[21]_a}^F(t)$ and $\psi_{[21]_a}^S(s)$ respectively. The S_3 Clebsch-Gordan coefficients $C_{[21]_a, [21]_a}^{[3]}$ take the values $1/\sqrt{2}$ for the two mixed symmetry components (112)

($a = 1$) and (121) ($a = 2$) respectively. The 3rd components of the isospin of the proton and the neutron are denoted t and the corresponding spin components s .

B. The $qqqq\bar{q}$ component

The possible $qqqq\bar{q}$ components of the nucleons fall into 2 classes: those, in which the $qqqq$ subsystem is in the P -state and the \bar{q} is in the ground state, and those, in which the $qqqq$ subsystem is in the ground state and the \bar{q} is in the P -state [10]. A “pion cloud” configuration may be represented by the latter, as the antiquark in the pseudoscalar pion has to be in a P -state. Which one of the $qqqq\bar{q}$ configurations that has the lowest energy, and consequently the largest amplitude in the proton, will depend on the form of the hyperfine interaction between the quarks.

If the hyperfine interaction depends on spin and/or isospin, the lowest energy configuration will be that with the most antisymmetric spin and/or isospin symmetry, as those yield the largest matrix elements of the operators $\sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j$, $\sum_{i<j} \vec{\tau}_i \cdot \vec{\tau}_j$ and $\sum_{i<j} \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$. In the case, where the \bar{q} is in the ground state, the lowest energy configuration is that, in which the $qqqq$ subsystem has the isospin-spin symmetry $[4]_{FS}[22]_F[22]_S$. When the \bar{q} is in the P -state the $qqqq$ subsystem with the mixed flavor-spin symmetry $[31]_{FS}[22]_F[31]_S$ has the lowest energy.

1. The antiquark in the S -state

The $qqqq\bar{q}$ state with the lowest energy, for which the \bar{q} is in the ground state, has the $qqqq$ subsystem in the P -state, with the mixed spatial symmetry $[31]_X$. The wave function for this 5-quark system may be expressed in the form [11]:

$$\begin{aligned} \psi_N(t, s)^{5q} = & \sum_{a,b} \sum_{m,\bar{s}} (1, \tfrac{1}{2}, m, \bar{s} | \tfrac{1}{2}, s) C_{[31]_a[211]_a}^{[1^4]} C_{[22]_b[22]_b}^{[4]} \\ & \psi_{[211]_a}^C \varphi_{[31]_a,m}(\{\vec{\xi}_i\}) \psi_{[22]_b}^F \psi_{[22]_b}^S \bar{\psi}^C \bar{\chi}_{t,\bar{s}}. \end{aligned} \quad (4)$$

Here the sum over a runs over the 3 configurations of the $[211]_C$ and $[31]_X$ representations of S_4 , and the sum over b runs over the 2 configurations of the $[22]$ representation of S_4 respectively [12]. The S_4 Clebsch-Gordan coefficients $C_{[31]_a[211]_a}^{[1^4]}$ take the values [12]:

$$C_{[31]_1[211]_1}^{[1^4]} = \frac{1}{\sqrt{3}}, \quad C_{[31]_2[211]_2}^{[1^4]} = \frac{-1}{\sqrt{3}}, \quad C_{[31]_3[211]_3}^{[1^4]} = \frac{1}{\sqrt{3}}. \quad (5)$$

The values of the S_4 Clebsch-Gordan coefficients $C_{[22]_b[22]_b}^{[4]}$ are $1/\sqrt{2}$ for $b = 1, 2$. In (4) the color and spin-isospin wave functions of the \bar{q} are denoted $\bar{\psi}^C$ and $\bar{\chi}_{t,\bar{s}}$ respectively.

The spatial wave functions $\varphi_{[31]_a,m}$ for the complete $qqqq\bar{q}$ system (4) will be products of symmetric function of the Jacobi coordinates $\vec{\xi}_i$, $i = 1, ..4$ (1) and one of the 3 vectors $\vec{\xi}_a$, $a = 1, 2, 3$, if the position coordinate of the \bar{q} is taken to be \vec{r}_5 .

2. The antiquark in the P – state

If the \bar{q} antiquark is in the P –state, the $qqqq$ subsystem is in the symmetric ground state. In this case its spin-isospin state has to have the mixed symmetry $[31]_{FS}$ in order to combine with the mixed symmetry color state $[211]$ to a completely antisymmetric state. The lowest energy spin-isospin configuration will in this case be $[31]_{FS}[22]_F[31]_S$. The corresponding complete wave function may be written in the form:

$$\begin{aligned} \tilde{\psi}_N(t, s)^{5q} = & \sum_{a,b,c} \sum_{m,m',\bar{s}} (1, \frac{1}{2}, m', \bar{s} | j, m) (j, 1, m, S | \frac{1}{2}, s) C_{[31]_a[211]_a}^{[1^4]} C_{[22]_b[31]_c}^{[31]_a} \\ & \psi_{[211]_a}^C \varphi_{[4],m'}(\{\vec{\xi}_i\}) \psi_{[22]_b}^F \psi_{[31]_c(S)}^S \bar{\psi}^C \bar{\chi}_{t,\bar{s}}. \end{aligned} \quad (6)$$

In this case the spatial wave function $\varphi_{[4],m}$ will be a symmetric function of the Jacobi coordinates $\vec{\xi}_i$, $i = 1, ..4$ multiplied by $\vec{\xi}_{4,m}$ (1).

In (6) the 3rd component of the spin of the $qqqq$ subsystem takes the values 1,0 and -1. In the case of the elastic nucleon form factors contributions may arise from the terms with $j = 1/2$ and $j = 3/2$.

3. The spatial wave function models

The spatial wave function model for the $qqqq\bar{q}$ state, for which the spatial wave function of the $qqqq$ subsystem has the mixed symmetry $[31]_X$ the following algebraic form will be employed

$$\varphi_{[31]_a,m}(\{\vec{\xi}_i\}) = \mathcal{N}_{[31]} \frac{\xi_{a,m}}{(1 + \frac{(\sum_{j=1}^4 \xi_j^2)}{2B^2})^{(A+1)}}, a = 1, 2, 3. \quad (7)$$

Here $\mathcal{N}_{[31]}$ is a normalization constant. This wave function has the appropriate threshold behavior for a P –state wave function [4]. The analogous wave function for the $qqqq\bar{q}$ state, for which the spatial wave function of the $qqqq$ subsystem is symmetric, and where the \bar{q} is

in the P -state, is taken to be

$$\varphi_{[4],m}(\{\vec{\xi}_i\}) = \mathcal{N}_{[4]} \frac{\xi_{4,m}}{(1 + \frac{(\sum_{j=1}^4 \vec{\xi}_j^2)}{2B^2})^{(A+1)}}. \quad (8)$$

The symmetric function of Jacobi coordinates in eqs. (7) and (8) leads to the fact that $\mathcal{N}_{[4]} = \mathcal{N}_{[31]}$.

C. Breit frame wave functions

In a covariant calculation the elastic nucleon form factors in instant form kinematics are most conveniently calculated as matrix elements of the initial and final wave functions in the Breit frame. For this the rest frame wave functions described above have to be boosted to the Breit frame, in which there is zero energy transfer to the nucleon.

Without loss of generality the momentum transfer to the nucleon may be taken to define the z -axis. A generalization of the instant form boost relations of ref. [4] then leads to the following relations between the constituent momenta \vec{p}_i in the Breit frame and in the rest frame (primes denote final state momenta):

$$\begin{aligned} \vec{p}_{i\perp} &= \vec{k}_{i\perp} = \vec{k}_{i\perp}' = \vec{p}_{i\perp}', \\ p_{i\parallel} &= v_0 k_{i\parallel} + v_{\parallel} \omega_i, \\ p_{i\parallel}' &= v_0' k_{i\parallel}' + v_{\parallel}' \omega_i', \\ E_i &= v_{\parallel} k_{i\parallel} + v_0 \omega_i, \\ E_i' &= v_{\parallel}' k_{i\parallel}' + v_0' \omega_i'. \end{aligned} \quad (9)$$

Here the energy components are defined as

$$\begin{aligned} \omega_i &= \sqrt{\vec{k}_i^2 + m^2}, \quad \omega_i' = \sqrt{\vec{k}_i'^2 + m^2}, \\ E_i &= \sqrt{\vec{p}_i^2 + m^2}, \quad E_i' = \sqrt{\vec{p}_i'^2 + m^2}. \end{aligned} \quad (10)$$

In these relations m denotes the constituent mass and $v = \{v_0, \vec{0}_{\perp}, v_{\parallel}\}$ and $v' = \{v_0', \vec{0}_{\perp}, v_{\parallel}'\}$ the constituent boost velocities in the initial and final states. These satisfy the constraint $v^2 = v'^2 = -1$.

In instant form kinematics the boost velocities may be defined as [4]:

$$v_{\parallel} = -\frac{Q}{2 \sum_{i=1}^n \omega_i},$$

$$v'_{\parallel} = \frac{Q}{2 \sum_{i=1}^n \omega'_i}. \quad (11)$$

In the calculation of the nucleon form factors the Jacobian matrix of the transformations (9) are also required. In the present application the electromagnetic coupling shall be assumed to take place on only one of the constituents. If this is taken to be first of the constituents, these Jacobians take the following form for the systems of 3 and 5 constituents:

$$J_3 = \frac{\omega_2 \omega_3}{E_2 E_3} (1 - v_{\parallel} \frac{k_{1\parallel}}{E_1}), \quad (12)$$

$$J_5 = \frac{\omega_2 \omega_3 \omega_4 \omega_5}{E_2 E_3 E_4 E_5} (1 - v_{\parallel} \frac{k_{1\parallel}}{E_1}), \quad (13)$$

for the initial state coordinates. The corresponding expressions for the final state coordinates are obtained by replacement of the variables by the corresponding primed variables.

As the spin quantization axis is rotated by the boosts, there is in principle a need to take this into account by appropriate Wigner rotations of the spin variables. The numerical significance of the Wigner rotations has, however, been found to be but minor in the region of momentum transfer up to 10 GeV², which is relevant for form factors [4, 13]. The Wigner rotations will for reasons of simplicity therefore not be considered here .

III. THE NUCLEON FORM FACTORS

A. Definitions

The electric and the magnetic nucleon form factors may be defined as the following matrix elements of the electromagnetic current density operator [14]:

$$G_E(Q^2) = \sqrt{1 + \tau} \langle \frac{1}{2} | J_0 | \frac{1}{2} \rangle, \quad (14)$$

$$G_M(Q^2) = \frac{\sqrt{1 + \tau}}{\sqrt{\tau}} \langle \frac{1}{2} | J_x | -\frac{1}{2} \rangle. \quad (15)$$

Here η is defined as

$$\tau = \frac{Q^2}{4M^2}, \quad (16)$$

where M is the nucleon mass.

The form factors are obtained as integrals over the momenta in the Breit frame. As the wave functions depend on rest frame momenta the rest frame each wave function for the qqq and $qqqq\bar{q}$ subsystem in the integrand has to be multiplied by the square root of the

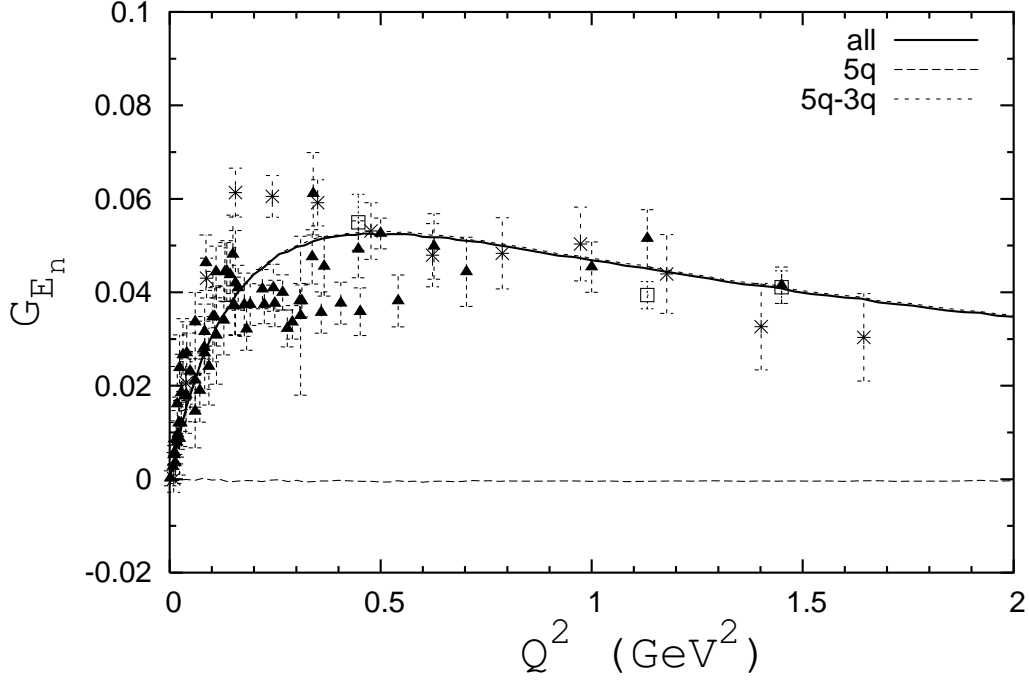


FIG. 1: Calculated neutron electric form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S -state. The long-dash curve represents the diagonal contribution from a $qqqq\bar{q}$ component with 3 % probability, and the short-dash curve the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are from ref. [15] (double crosses) [16] (boxes) and [17, 18] and references therein (triangles).

appropriate Jacobian - J_3 (12) or J_5 (13) for initial states or the corresponding ones with primed coordinates for the final states respectively.

The current operator of a constituent quark has the following matrix elements:

$$\langle \tfrac{1}{2} | J_0 | \tfrac{1}{2} \rangle = \sqrt{\frac{(E' + m)(E + m)}{4E'E}} \left\{ 1 + \frac{\vec{p}' \cdot \vec{p}}{(E' + m)(E + m)} \right\}, \quad (17)$$

$$\begin{aligned} \langle \tfrac{1}{2} | J_x | -\tfrac{1}{2} \rangle &= \frac{1}{2} \sqrt{\frac{(E' + m)(E + m)}{4E'E}} \left\{ \frac{|Q|(E' + E + 2m)}{(E' + m)(E + m)} \right. \\ &\quad \left. - \frac{(p_{\parallel} + p_{\parallel}') (E' - E)}{(E' + m)(E + m)} \right\}. \end{aligned} \quad (18)$$

The transition current operator for the transition $q\bar{q} \rightarrow \gamma$ has the corresponding matrix

TABLE I: The parameters of the nucleon wave function components.

m_q (MeV)	b_3 (MeV)	b_5	a_3	a_5
260	310	315	3.9	3.0

elements:

$$\begin{aligned} \langle \tfrac{1}{2} | J_0^{(a)} | \tfrac{1}{2} \rangle &= \frac{1}{2} \sqrt{\frac{(E' + m)(E + m)}{4E'E}} \left\{ \frac{|Q|(E' + E + 2m)}{(E' + m)(E + m)} \right. \\ &\quad \left. + \frac{(p_{\parallel} - p'_{\parallel})(E' - E)}{(E' + m)(E + m)} \right\}, \end{aligned} \quad (19)$$

$$\langle \tfrac{1}{2} | J_x^{(a)} | -\tfrac{1}{2} \rangle = \sqrt{\frac{(E' + m)(E + m)}{4E'E}} \left\{ 1 - \frac{p_{\parallel} p'_{\parallel}}{(E' + m)(E + m)} \right\}. \quad (20)$$

B. The electric form factors

1. The electric form factor of the neutron G_E^n

Consider first the electric form factor of the neutron G_E^n . As this form factor, with the present $SU(2)$ symmetric model wave function, takes no contribution at all from the diagonal transitions between the qqq component, the experimental values may be employed to set the amplitude of the $qqqq\bar{q}$ component in the neutron as well as the parameters in the wave function model phenomenologically.

The calculated neutron electric form factors are shown in Figs. 1 and 2 as obtained with a 3% probability for the $qqqq\bar{q}$ component, for both the cases where the \bar{q} is in the S -state and in the P -state, respectively. The wave function parameters employed in these calculations are listed in Table I.

These results reveal that the net contribution from the diagonal matrix elements of the $qqqq\bar{q}$ components in both cases is insignificantly small. The notable contribution arises from the transition matrix elements between the $qqqq\bar{q}$ and the qqq components. The results also show that there is little discrimination between the cases, where the antiquark is in the S - or in the P -states.

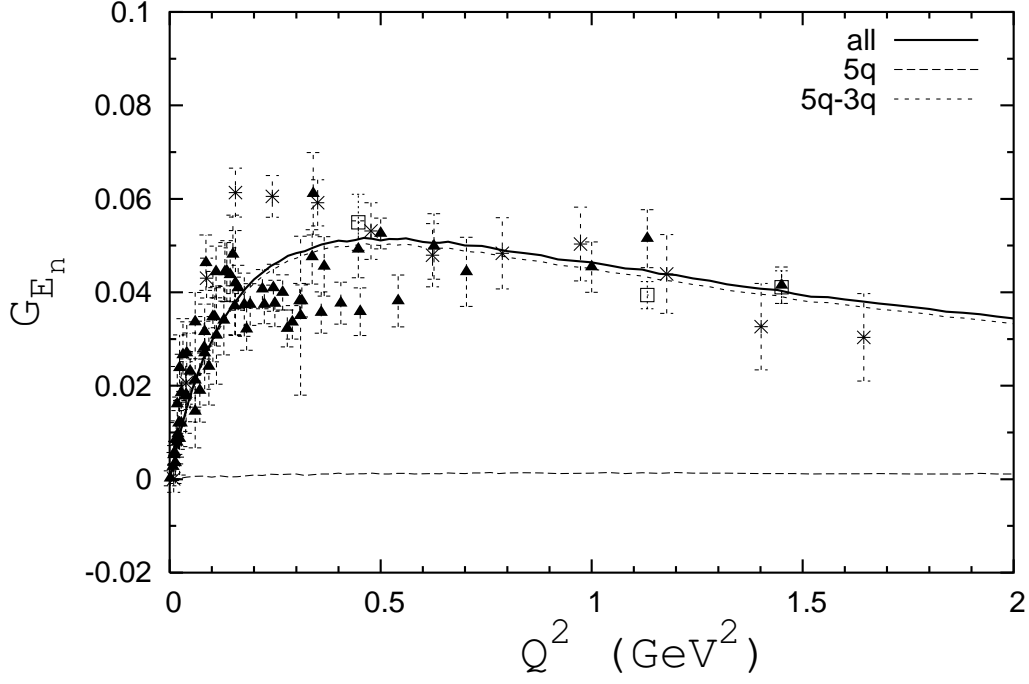


FIG. 2: Calculated neutron electric form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the P -state. The long-dash curve represents the diagonal contribution from a $qqqq\bar{q}$ component with 3 % probability, and the short-dash curve the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are the same as in Fig.1.

2. The electric form factor of the proton G_E^p

The calculated proton electric form factors are shown in in Figs. 3 and 4, again with a 3% probability for the $qqqq\bar{q}$ components for both the cases, where the \bar{q} is in the S -state and in the P -state respectively.

The results in Figs. 3 and 4 show that the contribution from the diagonal matrix elements of the $qqqq\bar{q}$ component are insignificant in comparison to that from the main qqq component. The $qqq - qqqq\bar{q}$ transition matrix elements are, however, significant, the significance growing with momentum transfer.

The sign of the transition matrix element contribution, which depends on the relative

sign of the qqq and $qqqq\bar{q}$ wave function components, is determined here by the sign of the contribution to G_E^n . The effect of including the $qqqq\bar{q}$ component on the calculated values for G_E^p leads to a much better description of the empirical form factor that is determined by polarization transfer, than what is possible without such a component [4]. The role of the $qqqq\bar{q}$ component is to bring about the desired faster falloff with Q^2 of G_E^p than of G_M^p . In the case of the conventional qqq quark model, the momentum dependence of the electric and magnetic form factors are very similar unless explicit interaction current contributions are considered.

In the case when the \bar{q} is in the P -state, the calculated electric form factor will have a zero between 10 and 11 GeV^2 in the impulse approximation. Such a zero cannot be achieved in instant form kinematics in the impulse approximation without additional wave function components beyond the basic qqq component. A similar zero does, however, appear naturally in the impulse approximation in front form kinematics already in the qqq model [4].

In Fig.5 a plot of the ratios of the calculated electric form factors of the proton to the phenomenological dipole form is given. These curves show that the two wave function configurations lead to different results only above 7 GeV^2 . Both curves fall close to the values that are extracted from the empirical cross section by the forward-backward separation method, once the correction from two-photon exchange has been accounted for [19, 20].

The electric mean square radii that correspond to these form factors are listed in Table II. In the case of the proton mean square radius the main contribution is that from the qqq component in the wave function. The transition matrix elements yield contributions that are smaller by an order of magnitude, while the diagonal matrix elements of the $qqqq\bar{q}$ components are insignificant.

In the case of the neutron, the main contribution arises from the transition matrix elements between the qqq and the $qqqq\bar{q}$ components. The net value for the mean square radius is very close to the empirical value, in the case where the antiquark is in the P -state.

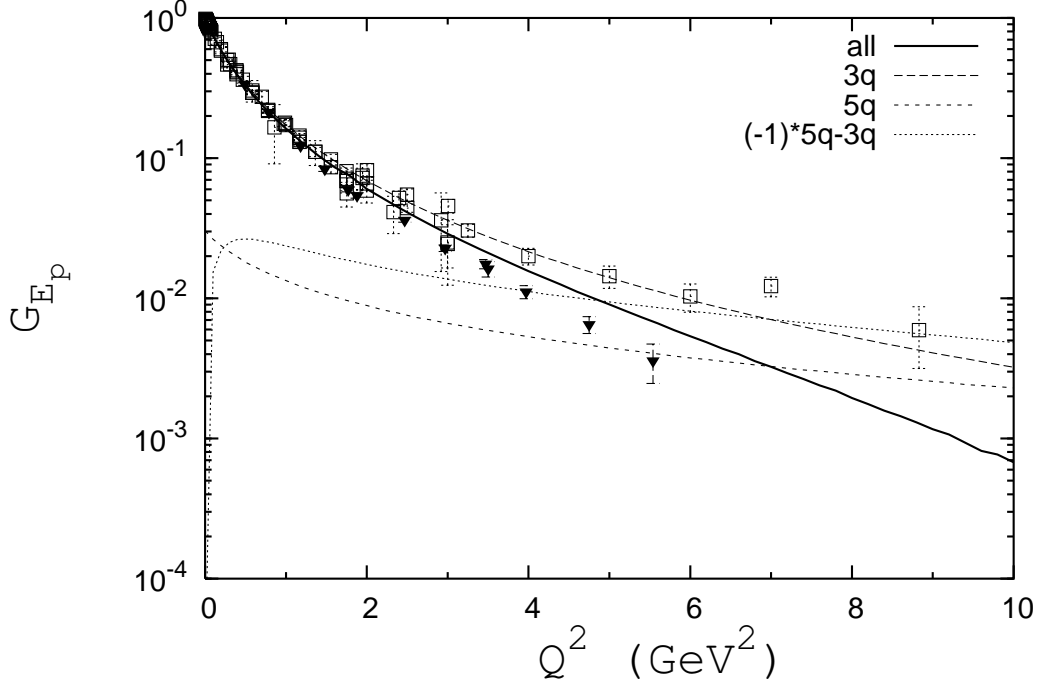


FIG. 3: Calculated proton electric form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S -state. The long-dash curve represents the diagonal contribution from the qqq component, and the short-dash curve that from the diagonal $qqqq\bar{q}$ component matrix elements with 3 % probability. The dotted curve represents the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are from ref. [21] and references therein.

C. The magnetic form factors

1. The magnetic form factor of the proton G_M^p

The calculated magnetic form factors of the proton are shown in Figs. 6 and 7. In the case where the \bar{q} is in the S -state there is no diagonal contribution from the $qqqq\bar{q}$ component. In the case, where the \bar{q} is in the P -state the contribution from the diagonal matrix elements of the $qqqq\bar{q}$ component is smaller by two orders of magnitude than that from the corresponding qqq matrix elements. The contribution from the $qqqq\bar{q}$ transition matrix elements is in both

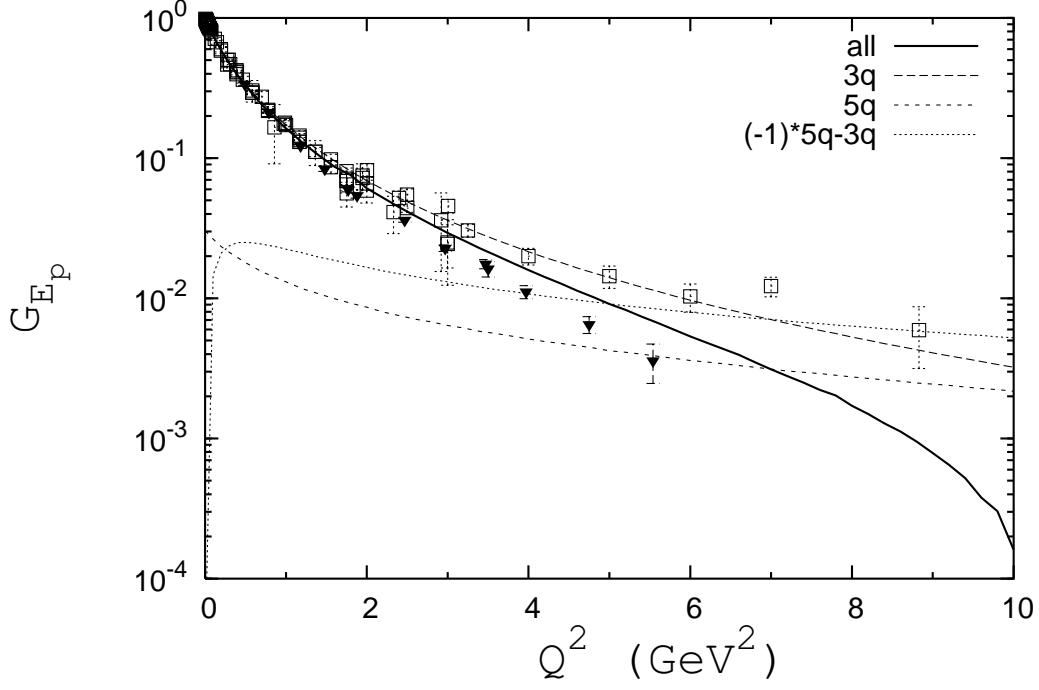


FIG. 4: Calculated proton electric form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the P -state. The long-dash curve represents the diagonal contribution from the qqq component, and the short-dash curve that from the diagonal $qqqq\bar{q}$ component matrix elements with 3 % probability. The dotted curve represents the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are the same as in Fig.3.

cases smaller by one order of magnitude than that from the diagonal matrix elements of the qqq components. The overall effect of the $qqqq\bar{q}$ components is small.

The corresponding magnetic moments are listed in Table III. While the diagonal contributions of the $qqqq\bar{q}$ components to these are insignificant, the transition matrix elements are substantial. As the magnetic moment contribution to the proton magnetic moment from the $qqqq\bar{q}$ component is smaller in the case, where the \bar{q} is in the P -state, that configuration is preferred. This configuration is the one, which admits an interpretation as a pion loop fluctuation.

In Fig.8 a plot of the ratios of the calculated magnetic form factors of the proton to

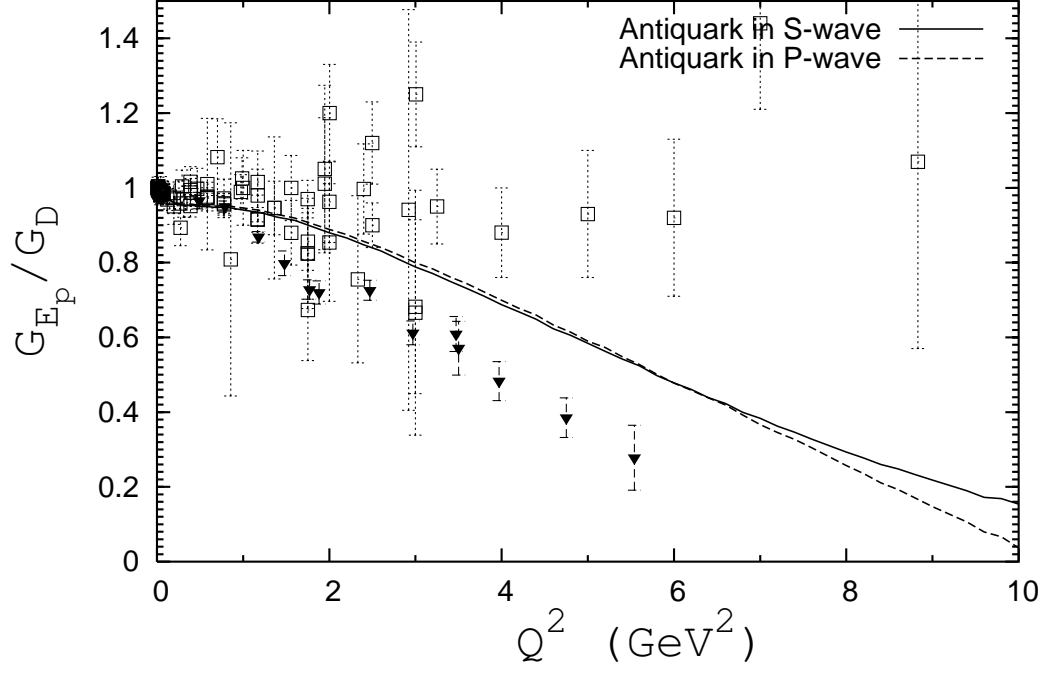


FIG. 5: Ratios of the calculated proton electric form factors to the dipole form in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S - (solid curve) and P -states (dashed curve). The data points correspond to those in Fig.3.

the phenomenological dipole form is given. These curves reveal a clear phenomenological preference for the wave function configuration, in which the antiquark is in the P -state. In that configuration the calculated magnetic form factor follows the empirical one fairly well over the whole range of momenta considered.

2. The magnetic form factor of the neutron G_M^n

The calculated magnetic form factors of the neutron are shown in Figs. 9 and 10. In this case the sign of the (small) diagonal $qqqq\bar{q}$ contribution depends on whether or not the \bar{q} is in the S - or in the P -state. The rate of falloff with momentum transfer is somewhat faster in the former case.

The non-diagonal contribution from the $qqqq\bar{q} - qqq$ transition matrix elements to the

TABLE II: The calculated nucleon electric radii.

		Proton	$\langle r_e^2 \rangle_{exp} = 0.774 \text{ fm}^2$		
\bar{q} state	qqq		$qqqq\bar{q}$	$qqqq\bar{q} \rightarrow qqq$	Total
S	0.675		0.009	0.052	0.736
P	0.675		0.005	0.049	0.730
		Neutron	$\langle r_e^2 \rangle_{exp} = -0.113 \text{ fm}^2$		
S	0.		-0.005	-0.103	-0.108
P	0.		-0.012	-0.098	-0.110

TABLE III: The calculated nucleon magnetic moments.

		Proton	$\mu_{exp} = 2.79\mu_N$		
\bar{q} state	qqq		$qqqq\bar{q}$	$qqqq\bar{q} \rightarrow qqq$	Total
S	2.51		0.0	0.45	2.97
P	2.51		0.01	0.25	2.77
		Neutron	$\mu_{exp} = -1.91\mu_N$		
S	-1.67		0.01	- 0.90	-2.56
P	-1.67		-0.02	-0.49	-2.18

neutron magnetic moments that are given in Table III are about twice as large in the case of the neutron as in the case of the proton. In this case there is a clear preference for the $qqqq\bar{q}$ configuration, in which the \bar{q} is in the P -state over that in which the \bar{q} is in the S -state. The large value for the magnetic moment in the latter case agrees with the corresponding value found in ref. [5] in the harmonic oscillator model when the spatial extents of the qqq and $qqqq\bar{q}$ configurations are equal.

While the value of the calculated magnetic moment of the proton is very close to the empirical value if the antiquark in the $qqqq\bar{q}$ component is in the P -state, the corresponding value of the magnetic moment of the neutron is too large by 14 %. This indicates that the model wave functions employed for the $qqqq\bar{q}$ are too crude to describe both the form factors and the static observables simultaneously. A reduction of the probability of the $qqqq\bar{q}$ component from 3% to 2% would reduce the overestimate by half, but at the price of

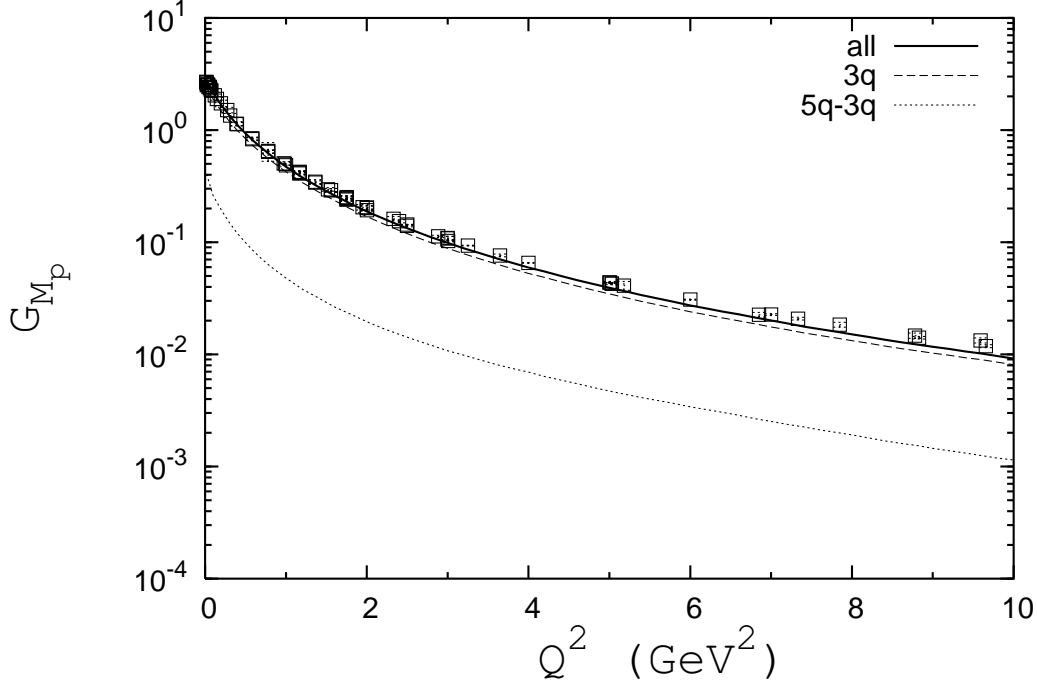


FIG. 6: Calculated proton magnetic form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S -state. The long-dash curve represents the diagonal contribution from the qqq component. The diagonal $qqqq\bar{q}$ component matrix elements with 3 % probability is insignificant and therefore not shown. The dotted curve represents the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are from ref. [21] and references therein.

a slightly less true description of the electric form factor of the neutron.

The calculated electric mean square radii of the neutron are listed in Table II. In this case the calculated values for both the wave function models are similar, which is to be expected, as the wave function parameters were chosen so that the calculated electric form factor of the neutron would follow the empirical shape.

In Fig.11 a plot of the ratios of the calculated magnetic form factors of the neutron to the phenomenological dipole form is given. These curves again show that the wave function configuration, in which the antiquark is in the P -state, leads to better agreement with the

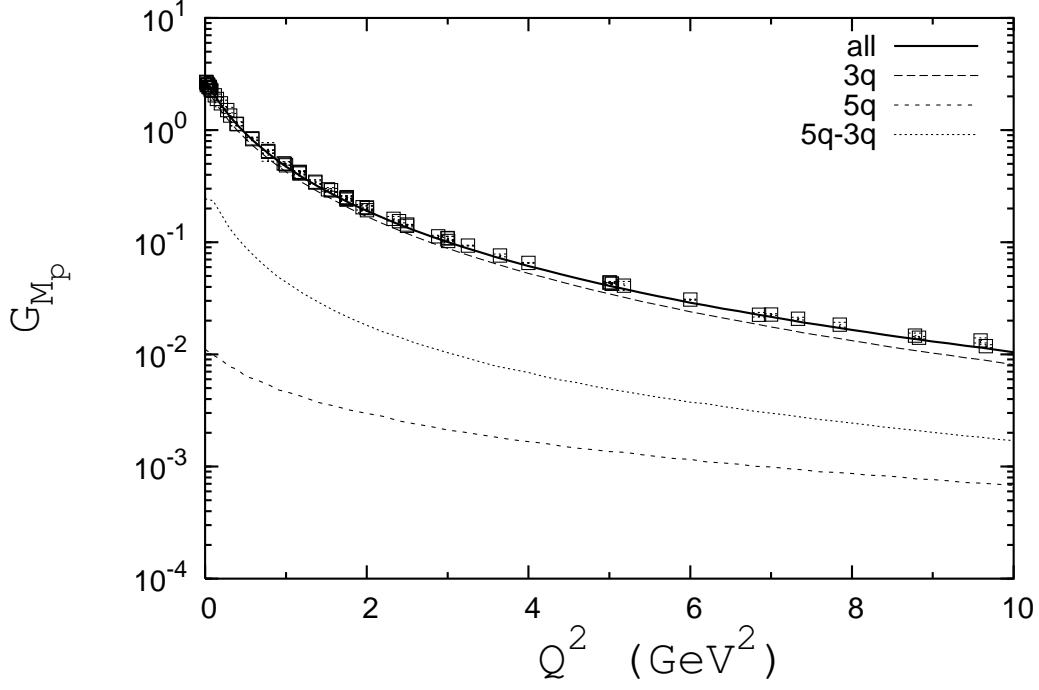


FIG. 7: Calculated proton magnetic form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the P -state. The long-dash curve represents the diagonal contribution from the qqq component, and the short-dash curve that from the diagonal $qqqq\bar{q}$ component matrix elements with 3 % probability. The dotted curve represents the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are from ref. [22] and references therein.

the empirical one over the range of momenta considered.

IV. DISCUSSION

The present exploratory study of the possibility to extend the the $SU(2)$ constituent quark model so that it provides a qualitative description of all the 4 electromagnetic form factors of the nucleon reveals that this is possible once a minor $qqqq\bar{q}$ component is included in the wave function. The results also indicate a clear preference for a $qqqq\bar{q}$ configuration, which has the same quantum number as a pion loop fluctuation (“pion cloud”). In the

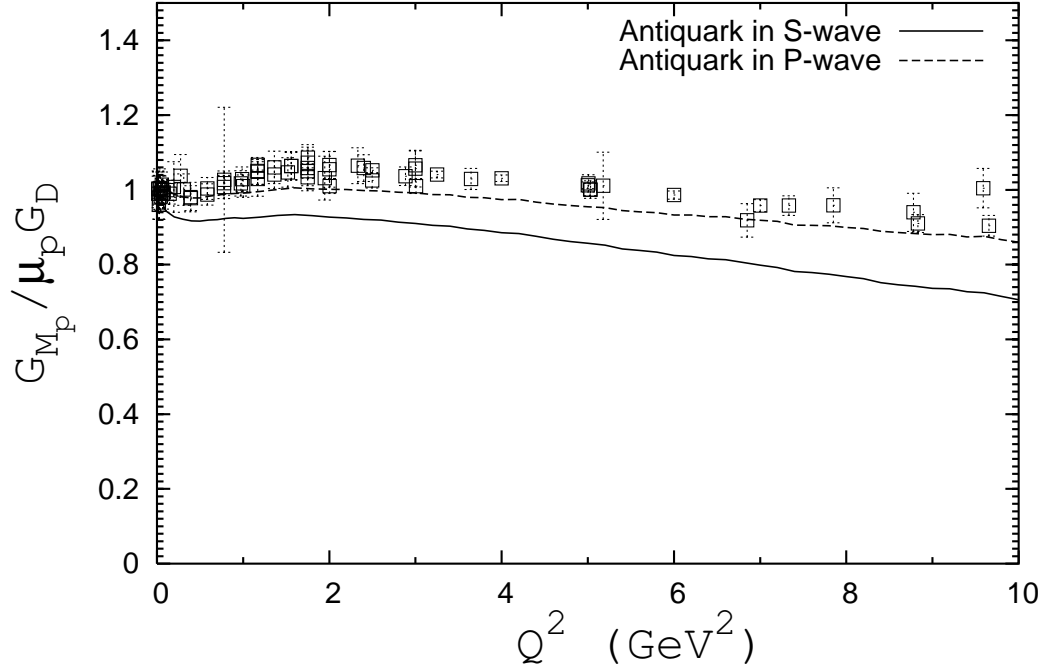


FIG. 8: Ratios of the calculated proton magnetic form factors to the dipole form in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S - (solid curve) and P -states (dashed curve). The data points correspond to those in ref. [22] and references therein.

present work, the spatial extent of the $qqqq\bar{q}$ component were taken to be similar to that of the main qqq component, even though a pion cloud configuration might in contrast be expected to be more extended than the main qqq component. The implication is that the form factors may not in the end be very sensitive to the range of the pionic wave function component.

In the present phenomenological study, the probability of the $qqqq\bar{q}$ component was determined by a fit to the empirical electric form factor of the neutron. As this form factor obtains no contribution from the qqq component (unless the very small contribution from the Wigner rotations are taken into account [4]), it is exceptionally sensitive to minor wave function components. There are several ways of providing a phenomenological description of this form factor. To these belong inclusion of a minor mixed symmetry S -state component and - as considered here - explicit sea-quark components. The reality is most likely to be a

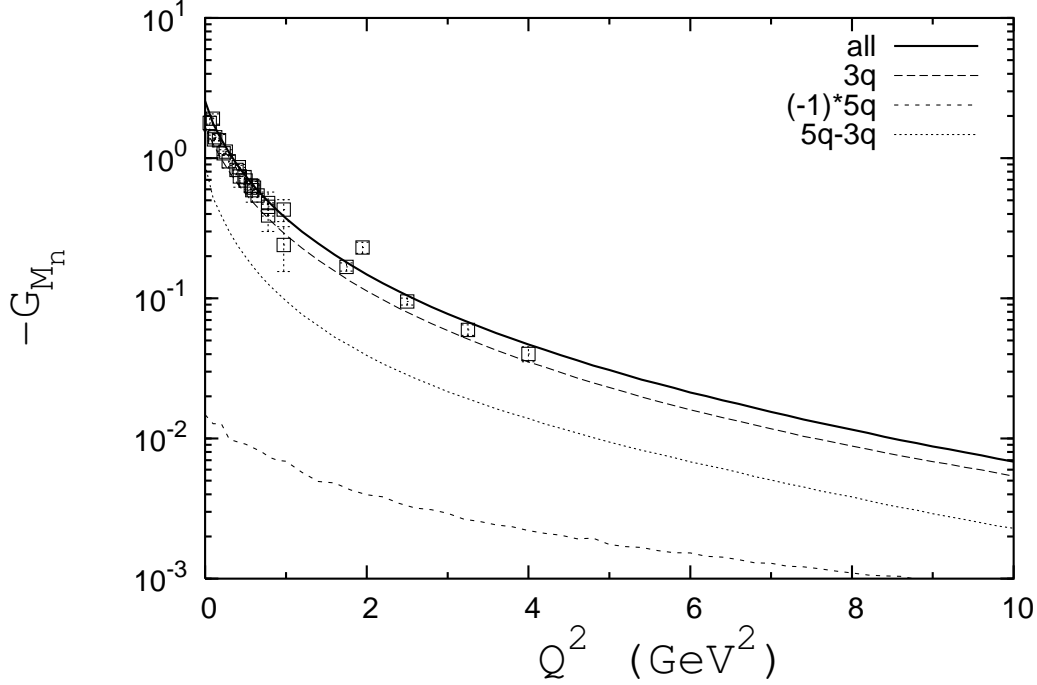


FIG. 9: Calculated neutron magnetic form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S -state. The long-dash curve represents the diagonal contribution from the qqq component, and the short-dash curve that from the diagonal $qqqq\bar{q}$ component matrix elements with 3 % probability. The dotted curve represents the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are from ref. [18, 22, 23] and references therein.

combination of several such mechanisms. The present finding that the electric form factor of the neutron can be described with a $qqqq\bar{q}$ component with $\sim 3\%$ probability is in any case physically intuitive.

In the present study the wave function parameters and the constituent mass were determined by a fit to G_E^n and the proton magnetic moment. With these parameters the qualitative description of the momentum dependence of the charge form factors of both the nucleons was achieved, as well as of the magnetic form factors. While the calculated electric radii of the nucleon and the proton magnetic moment are close to the corresponding

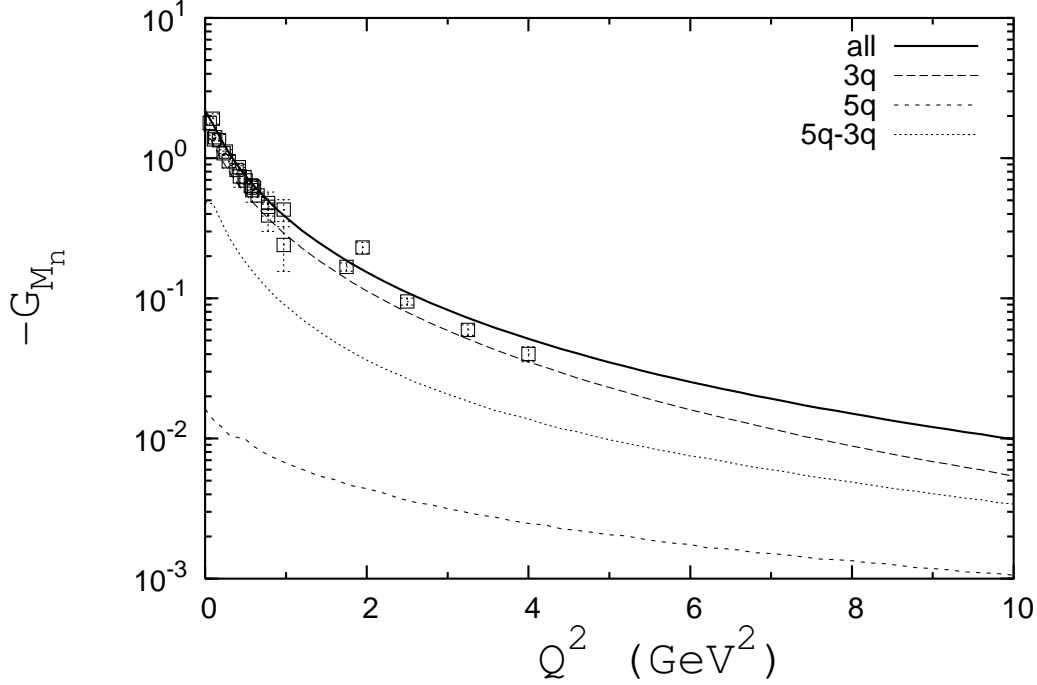


FIG. 10: Calculated neutron magnetic form factor in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the P -state. The long-dash curve represents the diagonal contribution from the qqq component, and the short-dash curve that from the diagonal $qqqq\bar{q}$ component matrix elements with 3 % probability. The dotted curve represents the corresponding contribution from the $qqq - qqqq\bar{q}$ transition matrix elements. The solid curve shows the combined result. The data points are from ref. [18, 22, 23] and references therein.

empirical values, the calculated magnetic moment of the neutron is too large. A more realistic value would need a combination of such $qqqq\bar{q}$ configurations, which lead to the same magnetic moment ratio for the nucleons as the conventional qqq wave functions.

The present results represent a non-exclusive alternative to the inclusion of mixed symmetry S - and D -state components in the wave function, which have previously been suggested as partial explanations for the electric form factor of the neutron [4]. As the presence of such wave function components are a natural consequence of spin-dependent hyperfine interactions between the constituent quarks, the best description may most likely be obtained

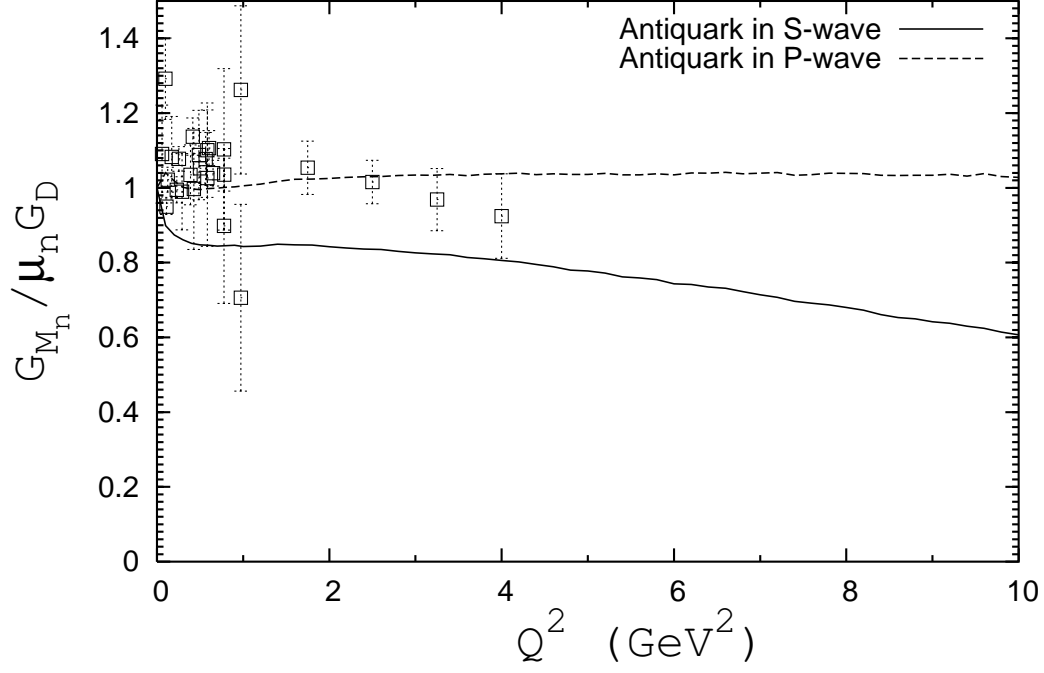


FIG. 11: Ratios of the calculated neutron magnetic form factors to the dipole form in the presence of a $qqqq\bar{q}$ contribution with the antiquark in the S - (solid curve) and P -states (dashed curve). The data points correspond to those in ref. [18, 22, 23] and references therein.

by a combination of such effects with sea-quark components of the form considered here.

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